# Biological Electric Fields and Rate Equations for Biophotons

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Ultraweak bioluminescence - the emission of biophotons - remains an experimentally well-established, but theoretically poorly understood phenomenon. This paper presents several related investigations into the physical process of both spontaneous biophoton emission and delayed luminescence. Since light intensities depend upon the modulus squared of their corresponding electric fields we first make some general estimates about the inherent electric fields within various biological systems. Since photon emission from living matter following an initial excitation ("delayed luminescence") typically does not follow a simple exponential decay law after excitation we discuss such non-exponential decays from a general theoretical perspective and argue that they are often to be expected and why. We then discuss the dynamics behind some nonlinear rate equations, connecting them both to biological growth rates and biophoton emission rates, noting a possible connection with cancer. We then return to non-exponential decay laws seen for delayed luminescence in an experimental context and again note a possible connection with cancer.

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## I. INTRODUCTION

Emission of biophotons from living cells and tissues of plant and animal origin are by now very well established (see, for example [1–4]). Emission of biophotons can be technically considered as a type of bioluminescence, although the observed emission of biophotons from biological tissues is much weaker than what is observed in normal bioluminescence and also different than thermal radiation emitted by tissues at their respective normal temperatures. There are two forms: a spontaneous one which occurs continuously and light emission following an initial optical excitation, often referred to as "delayed luminescence" (DL) which typically does not follow an exponential decay law. n this connection, we mention in passing that detailed studies of the associated (Poisson & sub-Poisson) statistics that arise from non-linear rate equations giving rise to them have also been made [8–10].

In Sec.(II), we discuss the biological cell and the nucleus within as cavities for resonating electromagnetic fields with the aim of investigation the idea that biophotons might initially reside in these cavities. Also, the frequencies and the magnitude of the mean electric fields are estimated. In Sec.(III), we discuss theoretical reasons for deviations from a linear rate equation and the resulting exponential decay law for fluorescence, which does not fit well with measured DL intensity as a function of time. In Sec.(IV), motivated by the logistic equation, we discuss non-linear rate equations and the dynamics generating them for photon emission by nonliving systems and make a connection to growth rates for size and mass of cells *etc.* In Sec.(V), the growth of a biological mass is analyzed and in Sec.(VI) connected to biophoton intensity and its correlation with the cancer cell growth is presented via some experimental results. In Sec.(VII), hyperbolic decays are considered and again a connection with cancer is noted. We close the paper with some concluding remarks in Sec.(VIII).

## II. ELECTRIC FIELDS AND FREQUENCIES IN BIO-SYSTEMS

Weak emission of biophotons (also known as ultraweak bioluminescence) from living cells and tissues of plant and animal origin is by now very well established [1–4] and occur not only in the ultraviolet, but also the visible region of the spectrum, but it's precise physical origins remain unclear and are the subject of ongoing investigation. There is an enormous literature, but recent reviews can be found in the monographs [5–7]. With no obvious excited molecules or atoms having been identified as being their sources, we estimate some of the characteristic frequencies one expects for natural biological cavity resonators.

Let $L$ denote the length scale of a cavity containing a fundamental frequency $\omega$ and a dielectric constant $\varepsilon$. These are related by

$$\omega = \left(\frac{\pi c}{L\sqrt{\varepsilon}}\right). \tag{1}$$

For a typical biological cell and nucleus we have, respectively, that

$$L_{\rm cell} \sim 10^{-3}{\rm cm} \quad \text{and} \quad L_{\rm nucleus} \sim 10^{-4}{\rm cm}. \tag{2}$$

It is thereby expected that

$$\omega_{\rm cell} \sim \text{infrared} \quad \text{and} \quad \omega_{\rm nucleus} \sim \text{optical}. \tag{3}$$

It is interesting to note here the size of the electric fields associated with one photon,

$$\left(\frac{\varepsilon E^2 L^3}{4\pi}\right) = \hbar\omega = \left(\frac{\hbar\pi c}{L\sqrt{\varepsilon}}\right),$$

$$E = \frac{2\pi}{L^2}\sqrt{\frac{\hbar c}{\varepsilon^{3/2}}}\ . \tag{4}$$

A cavity photon located in the cell and nucleus, respectively, have associated electric fields

$$1 \text{ Gauss} \equiv 299.792458 \left(\frac{\text{volt}}{\text{cm}}\right), \tag{5}$$

$$E_{\text{cell}}^{1 \text{ photon}} \sim 3.5 \times 10^{-2} \text{ Gauss} \sim 1 \left(\frac{\text{kilovolt}}{\text{meter}}\right),$$

$$E_{\text{nucleus}}^{1 \text{ photon}} \sim 350 \text{ Gauss} \sim 0.1 \left(\frac{\text{megavolt}}{\text{meter}}\right). \tag{6}$$

While the photon frequencies of the biological cavity modes in the cell and in the nucleus are in agreement with experiments, the estimates of the electric fields here presented are lower than previously reported [2, 11, 12].

## III. DISCUSSION ON NON EXPONENTIAL DECAY LAWS

It is often thought that the luminescence following an initial excitation should be exponential, but this invariably comes from some circular reasoning: either "this is well-known to be the case" or "this is what one would expect from $dN/dt = -\lambda N$ where $N$ is the excited population". Deviations from exponential decay are actually quite well-known [13], though relatively absent from most textbooks. Notable exceptions are the books by Ballantine [14] and Merzbacher [15]. To quote from Merzbacher (*op. cit.* p513), "...the fact remains that the exponential decay law, for which we have so much empirical support in radioactive decay processes, is not a rigorous consequence of quantum mechanics but the result of somewhat delicate approximations". Among those approximations is that the initial state be coupled to a large number of final states with similar energies. For a treatment of systems decaying into small numbers of final states and the attendant failure of the exponential "law", see, for example [16]. Also of interest is [17].

In fact, there are many simple physical systems which instead display hyperbolic decay laws. Typical examples are those which involve the excitation of pairs in the medium, which then recombine to emit light. This naturally gives decay laws which one would expect classically to obey $dN/dt = -\lambda N^2$. Note that this is a purely classical result and does not require coherent effects between the excited states, which would also be expected to give the same decay law. For a review of condensed matter analogs of hyperbolic delayed luminescence (DL) in living things, see [18]. Interestingly, in systems like CdS, the hyperbolic delayed luminescence depends strongly on the size of the grains, and in the nematic liquid crystal 4-methoxybenzylidene-49-n-butylaniline (MBBA) it is present in the crystalline form, but disappears on melting [18]. In other words, the character of delayed luminescence is not simply a matter of chemical composition, but can depend strongly on the form the material takes. Interestingly, the trend seems to be towards higher DL in systems exhibiting higher degrees of structure, a fact which seems relevant for biological systems.

Approximately hyperbolic decay laws also arise in correlated many-soliton states [19] which, again, may be relevant for biological systems.

It should be noted on general grounds, strict exponential decay is impossible in quantum mechanics. Khalfin showed [20] as far back as 1958 that the Paley-Wiener theorem, together with analyticity, forbids exponential decay at large times.

There is also a simple physical argument. Going to the energy representation from the time representation one finds (*i.e.* taking a Fourier transform of the amplitude, which would be proportional to $\exp\left(-\frac{1}{2}\lambda t\right)$) a Lorentzian which immediately gives two problems: the tail goes to infinity, so that a system with an initially finite energy could be found to have arbitrarily large energy. Indeed, the Lorentzian (squared) does not have a finite integral, so is unacceptable as a probability distribution for energy on physical grounds.

There is a theorem [21] that if $P(t)$ is the probability that a system with a finite mean energy remains excited – that is, the survival probability – must satisfy $dP(t)/dt = 0$ at $t = 0$, a property shared by neither the exponential nor hyperbolic decays, so they must at best be approximations where they do seem to work. This can also be seen directly from the textbook "derivation" of exponential decay before any approximations are made.

Thus we see that a strictly exponential decay law fails at both large and short times.

For the hyperbolic decay law, the same normalization problem in the energy representation is clearly present: the integral of $1/(t-a)$ from zero to infinity is infinite and, again, we find that a strictly hyperbolic decay law is unphysical, so nonlinear rate equations of this type must be approximations.

Weron and Weron [22] have argued for a survival probability in general (for the cases where one might otherwise derive, with approximations, an exponential decay) of the form $exp(-t^\alpha)$ where $\alpha > 0$ and $0 < \alpha < 1$. It is even possible to have oscillations in decay [23].

Since one in general expects deviations from either exponential or hyperbolic decays, any and all experimental data are welcome – the form of a decay law can, and indeed must, be more complicated than a simple exponential or hyperbolic decay law, however well those models may fit data over a restricted time interval.

In the following sections we shall discuss some nonlinear rate equations that have been successfully employed for diverse biological systems.

## IV. DYNAMICS BEHIND SOME SIMPLE NON-LINEAR RATE EQUATIONS

Here we shall discuss some simple non-linear rate equations and the dynamical reasons behind them. The linear rate equation where the rate is proportional to the number itself leads of course to an exponential growth or an exponential decay. But as in all practical systems, some non-linearity is bound to be present giving rise to a non exponential in time behavior.

The best studied example is that of the laser. Here, if the mean photon number rate equation were linear, the number of laser photons would increase exponentially. Of course, that cannot be otherwise or we would need an infinite source of energy, hence there must be some dynamical mechanism to saturate the number. The solution to this problem was first given by Willis E. Lamb. The famous Lamb equation for light intensity $I(t)$ [24] may be written as

$$\frac{dI(t)}{dt} = +\nu[a - I(t)]I(t). \tag{7}$$

The second term on the right hand side of Eq.(7) arises dynamically through the creation and annihilation of two-photons at a time, just as the first term is related to the creation and annihilation of single photons. The parameter $a$ is called the pump parameter and its sign is crucial in determining the steady state value of $I$.

If $a \leq 0$, the steady state value of $I$ (determined by the vanishing of the left side of Eq.(7)) is $I_{SS} \to 0$. Physically, for negative pump parameters there is no laser activity. On the other hand though, for $a > 0$, $I_{SS} \to a$ and hence the laser intensity increases linearly with $a$.

Also, the innocent looking Eq.(7) has buried in it a (second order) phase transition wherein $a$ acts as the order parameter. This is easily seen by considering $I_{SS}$ as a function of $a$. $I_{SS}$ is continuous at $a = 0$ but its derivative is not.

A simple model for a plethora of physical processes such as mean photon number, intensity, mass growth, magnetization *etc.* is provided by analogs of the Eq.(7) where the parameters $\nu$ and $a$ have different physical significance and their signs do play a crucial role in determining the fate of that physical system.

As discussed previously in Sec.(II), the frequency of a mode in a biological cell is inversely proportional to the length in accordance with Eq.(1). If the cell geometry fluctuates via the length scale $L$, then the frequency of the photon oscillator will be modulated. Because of such a modulation, the cell cavity will emit or absorb two photons at a time (as in the dynamical Casimir effect) thus leading to the above rate equation.

Similar (non-linear) rate equations must exist for any living system (such as for its size, cell number *etc.*) where their initial growth may be rapid but eventually cease, resulting in a limiting value (such as the maximum size). In the following section, we shall discuss a concrete case of biophoton emission from soybeans as well as the rate equation governing the growth in mass of said soybeans.

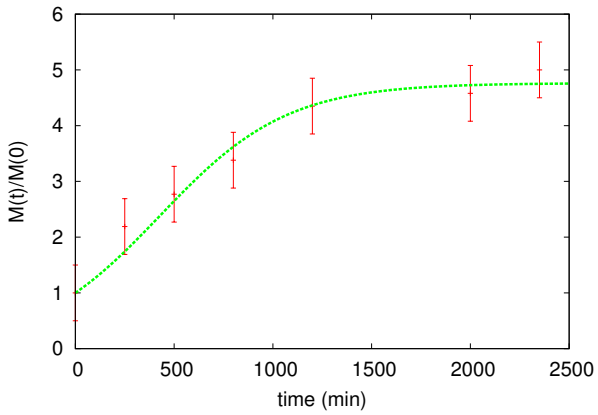


FIG. 1: Data points exhibited from F.A. Popp *et al.*' [2] showing the growing mass of soybeans as a function of time. The continuous curve is obtained using the logistic equation (Equation 9) and parameters given in Equation 10.

## V. GROWTH OF BIOLOGICAL MASS

The growth of the biological soybean sprouts plus roots has been written as the solution of the logistic differential equation[25, 26].

$$\frac{dM}{dt} = \nu\left(M - \frac{M^2}{M_\infty}\right) \tag{8}$$

In this model, for low mass the growth rate is proportional to mass since the nutrients feeding the bean sprout is proportional to the mass. For higher mass the loss rate from soybean waste emission is proportional to the square of the mass because of second order reaction kinetics.

With an initial mass $M_0$, the solution of Eq.(8) is given by

$$M(t) = \frac{M_0 M_\infty}{M_0 + (M_\infty - M_0)e^{-\nu t}}\ , \tag{9}$$

where $M_\infty$ is the final steady state value of $M(t \to \infty)$. In a measurement [2] of the growth of bean sprout mass, one had

$$\begin{aligned} \nu &= 3.1 \times 10^{-3}/\text{min}, \\ M_0 &= 1.3 \text{ gm}, \\ M_\infty &= 6.19 \text{ gm}. \end{aligned} \tag{10}$$

The comparison between the theoretical Eq.(9) and the experimental data on growing soybean sprouts is shown in FIG.1. The agreement between theory and experiment is satisfactory.

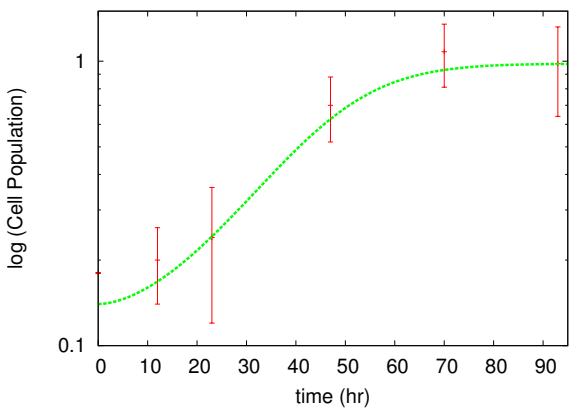


FIG. 2: Curve showing a stretched exponential curve (Eqs.(12) and (13)) with the experimental data of the growth of cancerous cell populations in arbitrary units as a function of time.

## VI. SPONTANEOUS BIOPHOTON EMISSION AND CANCER CELL GROWTH

It is well-documented that at high cell densities, large differences in the spontaneous ultraweak photon emission exist between normal cells and tumor cells originating from the same parental tissue [27, 28]. (In this section we confine our attention to the spontaneous biophoton emission and examine DL in section VII.) If the intrinsic rate of growth $\nu$ depends explicitly on time one finds a rate modulated logistic Eq.(8) of the form

$$\frac{dM}{dt} = \nu(t)\left(M - \frac{M^2}{M_\infty}\right). \tag{11}$$

The solution of Eq.(11) is

$$\eta(t) = \int_0^t \nu(s)ds,$$
$$M(t) = \frac{M_0 M_\infty}{M_0 + (M_\infty - M_0)e^{-\eta(t)}}\ , \tag{12}$$

The growth curves of the esophageal cancer cell (TE9) mass have been carefully studied [28]. We have fit the growth curves employing Eq.(12) with the stretched exponential form

$$\eta(t) = (t/\tau)^r, \quad \tau \approx 6.31 \text{ hr} \quad \text{and} \quad r \approx 1.75. \tag{13}$$

The results are plotted in FIG.2. The agreement with the stretched exponential form in Eqs.(12) and (13) is satisfactory.

The biophoton emission intensity from the cancer cells as a function of time has also been measured [28]. One may thus obtain the biophoton intensity as a function of the number of cancer cells. A plot of the results is shown in FIG.3. As the number of cancer cells increase, so does the biophoton emission rate of *each cell* (see, for example, Fig.(19.11) in [2]).

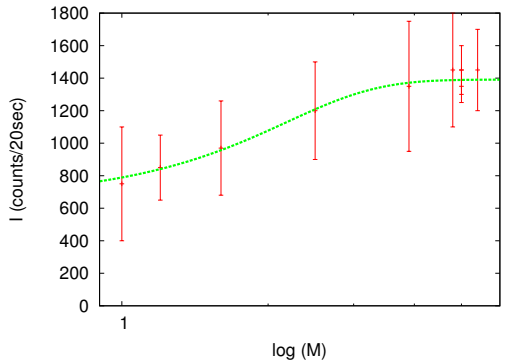


FIG. 3: Curve showing the theoretical fit of Eqs.(12) and (13) with experimental data from Takeda *et al.* The biophoton emission rate is shown as a function of the logarithm of the growing cancerous cell mass $M$ in arbitrary units.

## VII. DELAYED LUMINESCENCE

In this section we consider delayed luminescence and connect it with our earlier discussions.

A special case of Eq.(7) leads to a hyperbolic decay. Let us substitute

$$a = 0;\ \nu = \frac{\nu_o}{N(0)}, \tag{14}$$

so that Eq.(7) takes the form

$$\frac{dN(t)}{dt} = -(\frac{\nu_o}{N(0)})N^2(t), \tag{15}$$

whose solution is

$$N(t) = \frac{N(0)}{1 + \nu_o t} \tag{16}$$

A physical example [29] of DL decay obeying the hyperbolic law is provided in Fig.(4).

An extension of Eq.(16) to a fractional power in time, in analogy to that of the stretched exponential as in the last section [see Eq.(13)]

$$M(t) = \frac{M(0)}{[1 + \nu_o(t/t_o)^k]}, \tag{17}$$

has been made in [30], where light simulated biophoton reemission from normal and cancerous cells are compared. They find a considerable difference in the value of the parameter $k$ for the two cases.

## VIII. CONCLUSIONS

An important element in the analysis of biophotons concerns the magnitudes of the electric fields. Our estimates for the average electric fields in a cell and the nucleus - considered as a cavity - show that these fields

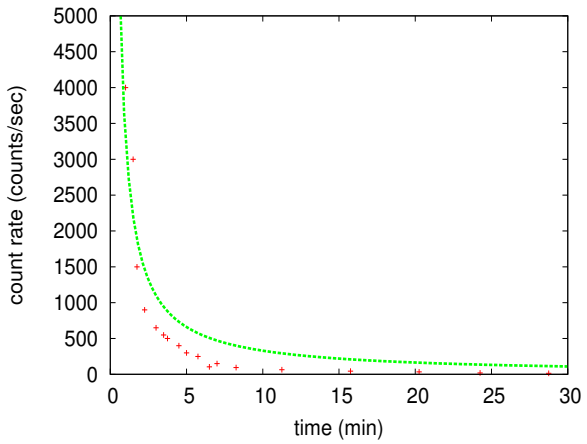


FIG. 4: Delayed luminescence intensity versus time for cells of *Bryophyllum daigremontanum*. The curve is the result of the fit of Eq.(16) with the experimental data.

are quite large, see Sec.(II). For example, the field due to one such biophoton in a nucleus is only slightly smaller than the breakdown field in humid conditions.

In the rest of the paper we have considered various rate equations of particular relevance for biological systems and provided physical reasons behind those equations where ever possible, see Sec.(IV), Sec.(V) and Sec.(VII). Theoretical arguments were presented to establish that the commonly employed exponential law for DL is not strictly tenable. A case of practical interest that is a delineation between the behavior of normal versus cancer cells is illustrated through differences in the values of parameters in their rate equations, see Sec.(VI) .


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